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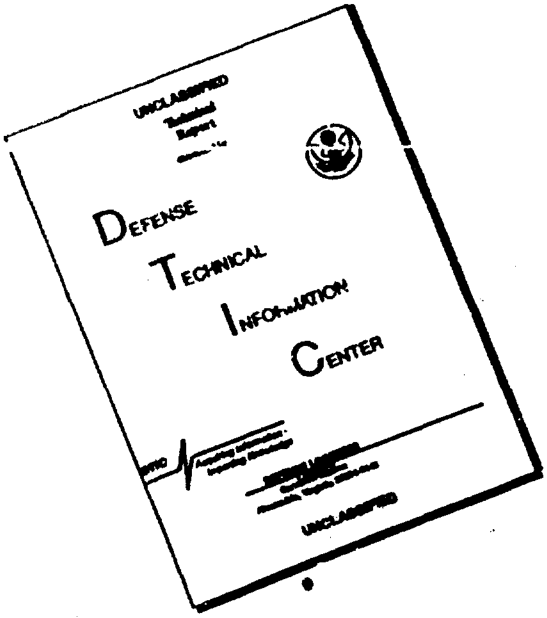

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FREE-FALL DECAY OF A MERCURY VAPOR PLASMA IN THE EARLY AFTERGLOW*

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A free-fall theory is developed for the decay of the average electron number density and of the electron temperature in a pulsed cylindrical mercury vapor discharge plasma using moment equations for the ions with a collision term for momentum loss, and assuming Maxwellian electrons. The plasma assumption was used. Good agreement is obtained with experimental results on four tubes of varying diameter.

As the mercury vapor pressure is lowered, the afterglow decay profile of the average electron density $\langle n_e(t)\rangle$ in a cylindrical discharge tube becomes quite nonexponential in time (Fig. 1). This Letter is concerned with the nonexponential behavior of $\langle n_e(t)\rangle$ which occurs when the mean free path of the charged particles is greater than the transverse dimensions of the tube. Experimental results are explained by using the first two moment equations for the ions, the plasma assumption, an ion-neutral momentum-loss collision term, the assumption that electrons are Maxwellian, and an equation describing the energy decay of the electrons.

The theory is based upon the good agreement between the simplified moment treatment of Kino and Shaw[1] and the more exact treatment of Self[2] and Parker.[3] We have used the steady-state initial conditions for electron density (= ion density) and drift velocity worked out in Ref. 1 for cylin-

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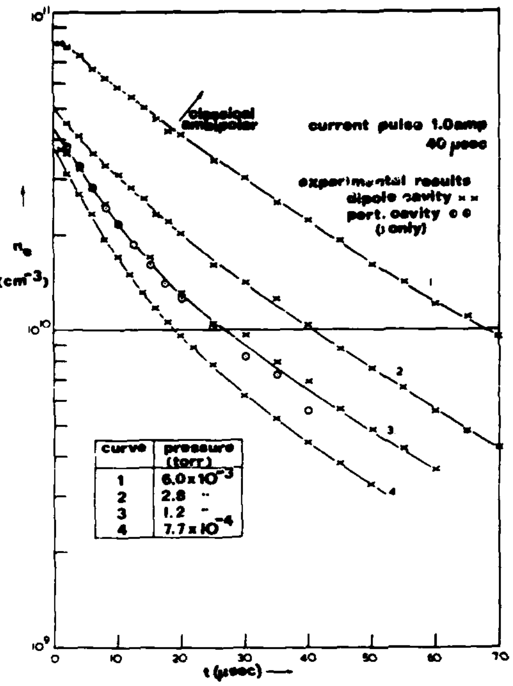

FIG. 1. Measured average electron number-density profiles in the afterglow. Tube C with pressure as parameter.

drical geometry, and (i) constant ionization with no ion kinetic pressure term, (ii) ionization proportional to electron density with no pressure term, and (iii) ionization proportional to electron density with an ion kinetic pressure term. Case (iii) was shown in Ref. 1 to give results very similar to the more exact treatment of Refs. 2 and 3, even though the fluid method assumes that all the ions have the same velocity at any point.

Our theoretical results for the afterglow indicate that in case (i) above the electron density decays about 20-30% more slowly than cases (ii) and (iii), which agree quite closely. Since case (iii) seems to be the most realistic here we use its results for the initial conditions in what follows.

In the afterglow we assume that the ion and electron densities are equal at all points in space and at all times. The ion kinetic pressure tensor term is dropped. We also assume that no further ionization occurs after $t = 0$. The electrons are assumed Maxwellian:

$$n_e = n = n_{e0} \exp(e\varphi/kT_e), \tag{1}$$

where n_e is the electron number density, n_{e0} its axial value, φ the space potential, $-e$ the electronic charge, k Boltzmann's constant, and T_e the electron temperature (assumed a function of time only and not of space). With these assumptions, the first two moment equations for the ions are

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rnv) = 0, \tag{2}$$

and

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{e}{M}\frac{\partial \varphi}{\partial r} + \nu v = 0. \tag{3}$$

Here v is the ion drift velocity, ν an ion-neutral collision frequency, and M the ion mass.

A third equation is necessary to describe energy changes. This states that the rate of decrease of average electron energy equals the electron wall flux times the energy equivalent of the wall potential:

$$\frac{d}{dt}\int_0^a \tfrac{3}{2} nkT_e 2\pi r dr = -2\pi a (vn)_a e\varphi_w, \tag{4}$$

where a is the tube radius and φ_w is the tube wall potential. This equation implies that each electron arriving at the wall transports energy $e\varphi_w$ from the electron gas, which is re-Maxwellianized immediately. The electron arrival rate at the wall equals that of the ions. In reality most electrons (energy less than $e\varphi_w$) will undergo numerous wall collisions during their lifetimes.

Since T_e is only a function of time, Eqs. (4) and (2) may be combined to give

$$\frac{dT_e}{dt} = (\tfrac{2}{3}\beta - 1)\frac{T_e}{N}\frac{dN}{dt}, \tag{5}$$

where $N = \int_0^a 2\pi n r dr$ and $\beta = e\varphi_w/kT_e$.

The fact that β is almost constant for a very wide variety of discharge parameters[2] enables us to integrate numerically the three coupled nonlinear equations (2), (3), and (5) starting with the initial n and v spatial distributions, case (iii) above. Constant collision frequency gives results which do not agree with experiment, and so a constant cross section for ion-atom collisions, σ, was assumed. The collision frequency is then $\nu = n_a \sigma v$, where n_a is the atom density. A typical set of curves with $kT_{e0} = 3.0$ eV and parameter $n_a\sigma$ is shown in Fig. 2.

The hot-cathode discharge tubes were described earlier.[4] Current pulses were of 0.2 to 2.0 A and from 5 to 50 μsec duration. Most results were

taken at 40 μsec, at which time steady-state conditions prevailed. The tubes were run in an oven at (100 ± 5)°C with a finger of liquid mercury protruding below into a water bath whose temperature was controlled to within ±0.2 C°.

The average electron number density was measured with a dipole resonance cavity[4] and a perturbation TM_{010} mode cavity. The two methods agree very well (see Fig. 1) when care is taken to account for the cavity end holes and the glass tube envelope.[5] The former method was used most of the time because the Q of the dipole-mode cavity is higher, yielding superior time resolution.

Electron temperature decay curves in these rapidly changing plasmas are exceedingly difficult to measure. We have used a pulsed Langmuir probe technique, a swept probe technique, and a time-resolving (1-2 μsec) 3.0-GHz radiometer.[6] However, at the pressures of interest here the absorptivity of the plasma is so low that the last method is rather insensitive. We chose to use the radiometer to verify the probe results at higher pressures. Even so, we do not place much confidence in our probe results at very low pressures.

Electron density profiles were extremely reproducible, even after periods of months. The shape of the profile did not depend on the discharge current pulse, but only on pressure and initial electron temperature. For tubes of different radii but with roughly similar initial conditions, approximate time constants of the $\langle n_e(t)\rangle$ profiles scaled directly as the tube radii, as might be expected for a free-fall regime.

To compare theory and experiment we proceed as follows: (i) Choose an initial electron temperature (usually measured) and select initial density and velocity space profiles. We have used those corresponding to free-fall conditions as described above, but collision effects could be accounted for here using the theories of Self and Ewald[7] and Forrest and Franklin.[8] (ii) Calculate $n_a\sigma$. Alternatively this may be selected to best fit experimental data. (iii) Solve Eqs. (2), (3), and (5) to obtain $\langle n(t)\rangle$ and $kT_e(t)$.

Figure 3 shows such a comparison for two tubes. Agreement for $\langle n(t)\rangle$ is excellent, but for $kT_e(t)$ somewhat poorer. As stated above, we are not very confident of the experimental electron temperature points, especially later in the afterglow.

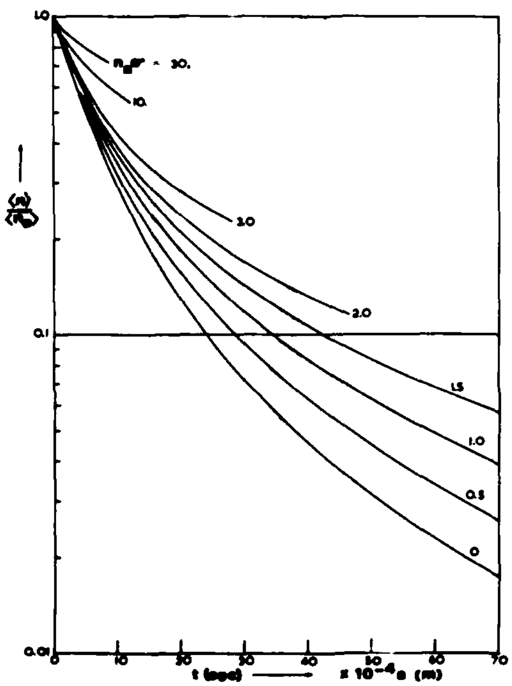

FIG. 2. Theoretical normalized average electron number density. Parameter $n_a\sigma$. kT_{e0} = 3.0 eV.

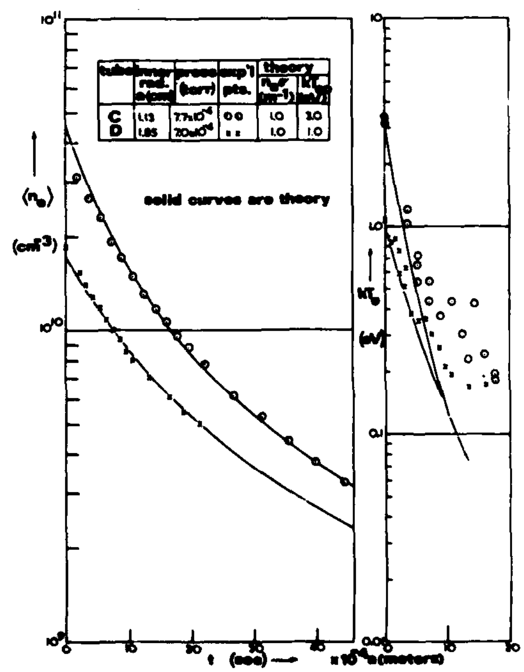

FIG. 3. Comparison of experimental electron density and temperature profiles in the afterglow for two tubes (radii 1.13 and 1.85 cm).

The parameter σn_a used to best fit theory with experiment yields $\sigma \approx 5\times10^{-20}$ m^2, a value of the order of the elastic ion-atom cross section but at least an order of magnitude smaller than the average charge-transfer cross section.[9] This is not yet understood fully. Possibly the introduction of the ion kinetic pressure term in the afterglow theory will clarify this point.

Using this theory with experimentally determined initial electron temperatures, we were able to obtain excellent agreement with electron density measurements made on four discharge tubes of radii 0.55, 0.85, 1.13, and 1.85 cm. More complete results will be presented elsewhere. This work tends to support the validity of using fluid theory for this sort of problem. It also predicts supercooling of the electrons[10]; whether or not this is real or a phenomenon introduced by approximations in the theory will be explored.

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